THE ACTUAL MASS OF POTENTIAL ENERGY, A CORRECTION TO CLASSICAL RELATIVITY*

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1. *Relativity and Potential Energy.*—Einstein's relation between mass and energy is universally known. Every scientist writes

$$E = Mc^2 \tag{1}$$

but almost everybody forgets to use this relation for potential energy. The founders of Relativity seemed to ignore the question, although they specified that relation (1) must apply to all kinds of energy, mechanical, chemical, etc. When it comes to mechanical problems, the formulas usually written contain the mass of kinetic energy, but they keep silent about the mass of potential energy. We must investigate this situation carefully and try to understand what sort of difficulties are raised by such a revision.

Let us consider a *physical body*, which we assume to be a closed structure, with an isolating boundary letting no energy trespass. It contains a certain energy E_0, that we may measure in a frame of reference where the body stays at rest. The internal energy may be chemical, mechanical, kinetic, or potential; it will change all the time from one type to another type; we state that this energy E_0 yields a rest-mass M_0 according to equation (1).

When the physical body is in motion with a constant velocity $\mathbf{v}$, we obtain a new mass M, with an energy E, and a momentum $\mathbf{p}$:

$$E_0 = M_0c^2 \quad E = Mc^2 \quad \mathbf{p} = M\mathbf{v}$$

$$M = \frac{M_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}. \tag{2}$$

The change from M_0 to M accounts for the mass of kinetic energy.

The physical body may be moving in a static field of forces and obtain, at a certain instant of time, an external potential energy U. Everybody assumes the total energy to be represented by the formula

$$E_{\text{tot}} = Mc^2 + U, \tag{3}$$

where U remains unchanged, despite the motion of the body at velocity v; this fact reveals that *one completely ignores any possibility of mass connected with the*

external potential energy. If this external potential energy had any mass, this mass would somehow be set in motion by the displacement of the physical body, and this moving mass would obtain some kinetic energy. No provision for any such effect can be seen in equation (3).

We are thus in a strange situation, where the internal potential energy obtains a mass, while the external potential energy does not! The contradistinction is striking and shocking! The discussion offered in the following sections will give its full weight to the preceding remarks.

2. *The Meaning of Potential Energy in Relativistic Theories.*—The definition of *potential energy* plays a prominent role in classical mechanics, but when we turn to Relativity, this quantity is high on the list of needed reappraisals. The original classical definition cannot be maintained, since it is based on "absolute time" and "infinite velocity of propagation" for signals. Many other definitions are in trouble for similar reasons: the third principle of Newton (equal *action and reaction* at any distance) and the notion of center of masses, etc.

How could we speak of equal action and reaction between the sun and the earth, for instance, when it takes about 8 min for a signal to propagate from one to the other? In 8 min, the earth travels quite a distance, and the attraction of the sun is modified. If an explosion occurs on the sun, its action will be felt on the earth 8 min later, and the reaction on the sun will come back 16 min later! The problem of the reliability of potential energy definitions is actually a very acute one.

There are other difficulties raised by Relativity in the definition of moment of momentum, or of moment of inertia, and more generally in the discussion of all *problems involving rotations,* that should be carefully re-examined; it is not certain that the necessary revisions have been actually performed correctly; this, however, is another story.

Let us concentrate on problems of potential energy. There must be a way out of the trouble, because we know that *Relativity joins smoothly with classical mechanics* when the following conditions are fulfilled:

(a) All velocities v must be very small compared to the velocity c of light:

$$v \ll c. \tag{4}$$

(This condition involves using small potential energies.)

(b) Distances r must remain small, so that delays in the propagation of signals may practically be considered as negligible:

$$\frac{r}{c} \ll \tau, \tag{5}$$

where τ is a characteristic time interval for the motion under consideration, e.g., its period. In the problem of sun and earth interaction, the first condition (a) is nearly fulfilled (except in Michelson's experiments), but the second condition (b) is not.

What must now be done is to investigate carefully a type of definition that can be used for a relativistic quantity which could replace potential energy, and reduce to potential energy in classical mechanics. We shall then be in a position to examine the space distribution of the new quantity and of the corresponding mass.

Before we discuss this problem we must consider another difficulty, resulting from traditional methods of classical mechanics. Many of these methods cannot be extended to Relativity, and finally also had to be abandoned in Quantum theories. Classical mechanics, with its *absolute time*, can state and discuss problems with any number of particles (say: M_1, M_2, ... M_n) located, at a certain instant t of absolute time, at r_1, r_2, ... r_n. The potential energy is supposed to be any function $U(r_1, r_2, \ldots r_n)$, and the problem is discussed in a mathematical space with $3n$ dimensions. Most theorems of classical mechanics are stated in this very general way.

Such a method is not applicable to relativistic problems, where each particle (coord. $x_n\, y_n\, z_n$) obtains its individual time t_n in a given frame of reference; Relativity is characterized by the use of a four-dimensional space-time.

The change in definitions is very serious and its consequences are many. For instance, let us consider a system of two particles interacting together: shall we state that potential energy is located on the first particle? Should it be attributed to the second one? Or split between them? If *energy means mass*, where shall we locate the mass? This is a fundamental question which we have to discuss.

The question has very often been ignored, or evaded, because it does not always appear clearly in all problems. One of the two bodies interacting may be very much heavier than the other one, hence almost motionless, e.g., the earth attracting Newton's apple! Newton carefully stated his third principle: the apple, too, is attracting the earth! But many theoreticians forgot about it: the earth does not move (so they said), it creates a steady field of forces, and the apple is moving in this "given" field. As a result, these theories would assume no mass corresponding to potential energy, and write the total energy as in equation (2). The flaw is, however, obvious, and this is why the present discussion is needed.

3. *The Importance of Fields in Einstein's Theories.*—All these questions hang closely together; they are tightly interrelated and have been in the back of the mind of a great thinker like Einstein. He explained clearly that action at a distance being forbidden, one should rely entirely on actions transmitted step by step by fields propagating through space. The importance of field theory was definitely brought into the foreground. The ideas launched by Faraday and Maxwell were completed by relativistic discussions. Fields were assumed to have a real physical existence, even when they do not act on any moving particle and go on unnoticed. Such an assumption looks pretty much like metaphysics, but it plays a dominant role in relativistic problems.

There is no more any question of action and reaction at finite distances, but the law of equal action and reaction applies locally, at any given point *xyzt* in space-time.

The *field assumes a very complicated role:* it carries energy, momentum, Maxwell's tensions, etc., and we want to emphasize the fact that *the field itself carries a mass*.

This is the *situation* which we intend to discuss very carefully, since its full significance has been partly overlooked by many theoreticians of Relativity.

Let us start with a simple problem, on which there is general agreement. We consider a sphere of radius a, with a mass M_0 and an electric charge Q, that is distributed on the sphere's surface. In a frame of reference at rest, this charge Q generates an electric field F at a distance r

$$\mathbf{F} = \frac{Q}{r^2} \mathbf{r}^0, \tag{6}$$

where $\mathbf{r}^0$ denotes a unit vector in the $\mathbf{r}$ direction. This electric field obtains an *energy density* (ESCGS units)

$$\epsilon_{el} = \frac{1}{8\pi} |F|^2 = \frac{Q^2}{8\pi r^4}. \tag{7}$$

According to the fundamental rule (1), this corresponds to a *mass-density*

$$\rho_m = \frac{1}{8\pi c^2} |F|^2 = \frac{Q^2}{8\pi c^2 r^4}. \tag{8}$$

The energy-density (7) and mass-density (8) can be integrated over the whole space, around the sphere a, and yield

$$E_{el} = \frac{Q^2}{2a} \qquad M_{el} = \frac{Q^2}{2ac^2}, \tag{9}$$

where E_{el} is the total electric energy in the field, and M_{el} represents the total mass in the field around the sphere. The sphere may have another mass M_0 of internal origin and its global mass amounts to

$$M_g = M_0 + M_{el}. \tag{10}$$

When we write such a formula, we take into account the fact that equation (8) indicates a very high concentration of mass in the immediate neighborhood of the sphere, and we assume that this mass may (*as a first approximation*) be taken as located upon the sphere itself.
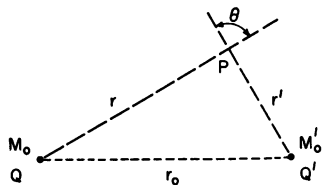

FIG. 1.

4. *Two Interacting Spheres.*—Let us go on with electric problems that are better known than many other similar ones and can be used as typical examples. We now select a two-body problem, with two spheres of radius a, rest masses M_0 and M'_0, charges Q and Q', supposed *at rest in a certain frame of reference;* we call r_0 the distance between them. Let us call P a point in space (Fig. 1) where we observe the resulting electric field

$$\mathbf{F} = \frac{Q}{r^2} \mathbf{r}^0 + \frac{Q'}{r'^2} \mathbf{r}'^0. \tag{11}$$

The *electric energy density* is now given by a formula

$$\epsilon_{el} = \frac{1}{8\pi} |F|^2 = \frac{1}{8\pi} \left[\frac{Q^2}{r^4} + \frac{Q'^2}{r'^4} + 2\frac{QQ'}{r^2 r'^2} \cos\theta \right], \tag{12}$$

where θ represents the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$.

The mass density becomes

$$\rho_m = \frac{\epsilon_{el}}{c^2} = \frac{1}{8\pi c^2} \left[\frac{Q^2}{r^4} + \frac{Q'^2}{r'^4} + 2\frac{QQ'}{r^2 r'^2} \cos\theta \right]. \tag{13}$$

In this remarkable formula, the first term obviously represents the contribution to the mass M_0 of the first particle, while the second term contributes to the M'_0 mass of the second particle, but *what is the meaning of the third term, with the QQ' cross product?*

In order to clarify this point, let us first consider the integral of the cross product in formula (12) for electric energy. We call ϵ_{int} the third term, that represents interaction between Q and Q'

$$E_{int} = \int_{-\infty}^{\infty} \epsilon_{int}\, d\tau = \frac{1}{4\pi} \int (\mathbf{F} \cdot \mathbf{F}') d\tau = -$$
$$\frac{1}{4\pi} \int \left(\frac{\delta V'}{\delta x} F_x + \frac{\delta V'}{\delta y} F_y + \frac{\delta V'}{\delta z} F_z \right), \quad (14)$$

where xyz are the coordinates of point P, while $d\tau$ is a volume element in space and $(\mathbf{F} \cdot \mathbf{F}')$ is the scalar product.

We introduced the static potential V' for charge Q', normalized by the usual condition $V' = 0$ at infinity:

$$V' = \frac{Q'}{r'}. \quad (15)$$

Integrating by parts, we find

$$E_{int} = -\frac{1}{4\pi} \left| V'(F_x + F_y + F_z) \right|_{-\infty}^{+\infty} + \frac{1}{4\pi} \int V'(\nabla \cdot F) d\tau.$$

The integrated term is zero and in the integral we note

$$(\nabla \cdot F) = 4\pi \rho_{el}, \quad (16)$$

where ρ_{el} is the electric density for charge Q. The result is

$$E_{int} = V'Q = \frac{Q'Q}{r_0}, \quad (17)$$

thence the following theorem:

The integrated interaction energy, taken over all space, yields the quantity usually called "potential energy" for two charges Q and Q', at rest in a certain frame of reference.

This means also that the total mass due to the cross product energy terms QQ' represents the mass of potential energy and is actually distributed in the whole space.

$$M_{pot} = \frac{Q'Q}{r_0 c^2}. \quad (18)$$

For two charges QQ' *at rest* in a certain frame of reference, we have been able to replace the mathematical abstraction of potential energy by a physical model, where the energy is distributed in space according to the field.

If we now want to discuss a problem of moving charges, we simply have to follow a similar procedure, and to compute the energy density in the field of both interacting particles. Terms in QQ' will yield directly the interaction energy, for any distance and any velocity. The energy distributed in space, according to the field, corresponds to mass.

Let us, for instance, consider a problem with one charge Q' at rest in a certain frame of reference, and the other mass moving with a velocity v. The field of Q' is the static field F' of equation (6), but the field F of the moving charge Q is represented by the well-known relativistic formulas [see, e.g., Sommerfeld, A., *Electrodynamics* (New York: Academic Press, 1952), p. 240, equation (14)].

Terms in QQ' in the energy density may then be computed (in this special frame of reference) together with the corresponding mass distribution. For high velocity v and large distance, some interesting results might be expected.

5. *Where Could the Mass of Potential Energy Be Localized?* Let us consider a problem where *conditions (4) and (5) are fulfilled*, and when we can speak of potential energy.

The mass of potential energy is actually distributed in the whole space, between and around the charges Q and Q'. If, however, we look more closely into the formula (13), we notice that the cross term (interaction)

$$\rho_{m,\text{int}} = \frac{QQ'}{4\pi c^2 r^2 r'^2} \cos \theta \tag{19}$$

is becoming very large on the charged spheres, when either $r = a$ or $r' = a$. This indicates a concentration of mass right on the two charges with much smaller density at a distance. The concentration, however, is not so strong as in equation (8): it goes as r^{-2} instead of r^{-4}. Nevertheless, we may introduce a first approximation similar to the one used in section 3 and state:

As a first approximation the mass of potential energy can be considered as localized in the interacting charges QQ' and split 50/50 between them. We rewrite equation (10) for the global masses in the following way:

$$\begin{cases} M_g = M_0 + M_{\text{el}} + \dfrac{QQ'}{2r_0 c^2} \\[2ex] M'_g = M'_0 + M'_{\text{el}} + \dfrac{QQ'}{2r_0 c^2} \end{cases} \tag{20}$$

The distribution of equation (19) is completely symmetrical in r and r' and this justifies the 50/50 split.

Some details, however, are worth discussing (Fig. 2). Formula (19) shows that the density of mass (and energy) obtains a certain sign at large distance, when θ is small and $\cos \theta$ is nearly unity. The $-$ or $+$ sign at large distance is given by the sign of the product QQ' and is the same as the sign in equation (18). However, we must notice that the $\rho_{m,\text{int}}$ density (19) is zero on a sphere C of diameter QQ', where we have $\theta = \pi/2$ and $\cos \theta = 0$. Within the sphere C, the density $\rho_{m,\text{int}}$ obtains an opposite sign.

Anyhow, the densities $\rho_{m,\text{int}}$ may have $+$ or $-$ signs, and (just as the potential energy itself) the mass of potential energy can be positive or negative.

The new masses (20), computed for particles at rest, are certainly a good first approximation when one of the particles moves at a low velocity v, since corrections should be only in v^2/c^2.

6. *Many Interacting Charges at Small Distances and Small Velocities.*—We discussed in some detail the case of two interacting electric charges Q and Q';

the results can be easily generalized to dipoles,
quadripoles, multipoles interacting with an
electric charge, or with some other multipoles.

Let us, for instance, consider a rigid struc-
ture at rest, holding a certain number of
charges Q', Q'', $\ldots Q^{(n)}$ and acting upon a
given charge Q. This may, for example, corre-
spond to the problem of a crystal lattice,
with a free electron Q moving through the
lattice. The charges Q', Q'', $\ldots Q^{(n)}$ may have
electric interaction between themselves, and this interaction will be part of the
total potential energy (and mass) of their rigid structure. The free charge Q may
interact with any one of the $Q^{(n)}$ charges, and half of the corresponding mass of
interaction is localized on Q, while the other half is on $Q^{(n)}$. Let us call U the
potential energy of all these interactions:

$$U = \sum_{j = 1.2..n} \frac{QQ(j)}{r_j}.$$ (21)

The mass of the free charge Q interacting with the structure becomes

$$M_Q = M_0 + M_{el} + \frac{1}{2c^2} U,$$ (22)

while there is an additional $\frac{1}{2c^2} U$ mass on the rigid lattice. This is a straight-
forward generalization of equation (20).

Let us now assume the charge Q to be moving with a small velocity v, the total
energy of particle Q plus lattice is

$$E_{tot} = \frac{M_0 + M_{el} + \frac{1}{2c^2} U}{\sqrt{1 - \frac{v^2}{c^2}}} c^2 + \frac{1}{2} U$$ (23)

instead of (3).

This can be rewritten in a slightly different way:

$$E_{tot} = \frac{M_0 + M_{el}}{\sqrt{1 - \frac{v^2}{c^2}}} c^2 + U + \left[\frac{U}{2} \left(\sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} - 1 \right) \right].$$ (24)

The last term, within the *brackets* is the *new term* corresponding to our theory,
as shown directly by a comparison of (24), (3), and (10).

In most practical applications this new term remains small, and Einstein's
equation (3) represents a good approximation. Our new correction might become
of importance only for large values of the velocity v and of the potential energy U.
A large velocity v would require special treatment as noticed at the end of section 4.
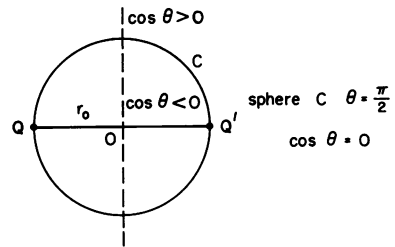According to the sign of U, the correction may be positive or negative.

Fig. 2.

The assumption that the new mass distribution is primarily located on the electric field in the whole space satisfies the obligation for relativistic transformations just as for the electromagnetic field itself. The simplified model with additional mass localized on the particle must be considered only as a simplifying approximation.

7. *Generalizations; Quantum Problems.*—The preceding method can be easily generalized to many other problems of "potential energy." The first step is to introduce a convenient type of field, propagating around each source. The next problem is to obtain the formula for the energy density, corresponding to equation (7), and then the calculation proceeds as in section 4. In such discussions, one should always beware of so-called "potentials," that are usually defined up to an arbitrary constant (or function), and directly lead to "gauge" troubles.

Quantum problems were discussed by W. Lamb, H. Bethe, J. Schwinger, and others, and their papers can best be found in Schwinger's book entitled *Quantum Electrodynamics* (New York: Dover, 1958). The method leads to corrections on the rest mass of particles, called "mass renormalization," and yields excellent numerical results. Quantum effects include electrostatic potential energy and all sorts of spin effects.

The present discussion proves that *mass-renormalization* is not only needed in quantum theories, but that it must already be introduced in classical Relativity, where it was completely overlooked by the founders of Relativity. Sommerfeld and Dirac were not aware of the difficulty, and their formulas must be very carefully revised. A first draft of the present paper was published in French.[1]

* Contract Nonr 266(56).

[1] Brillouin, L., "La masse de l'énergie potentielle," *Compt. Rend.*, **259**, 2361 (1964); Brillouin, L., "L'énigme *E* = *Mc²*: Energie potentielle et renormalisation de la masse," *J. Phys. Radium*, **25**, 883 (1964).

BICHROMOSOMAL SYNTHETIC SEMILETHALS IN DROSOPHILA PSEUDOOBSCURA*

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The mutation process is the ultimate source of the genetic raw materials from which evolutionary changes are compounded by natural selection. Without mutation, evolution would eventually be arrested. Populations of sexually reproducing, diploid, and polyploid organisms carry, however, enormous stores of potential genetic variability. This variability is gradually released by recombination. The release of the variability can be demonstrated experimentally. Populations of Drosophila carry many recessive lethal, semilethal, and subvital genetic variants, mostly concealed in heterozygous condition in "normally" viable individuals. Some of these lethals and semilethals arise by mutational changes in single genes, and perhaps by deletions of small blocks of genes. Other lethals and